HYBRID MODELING OF DIRECT AND INVERSE PROBLEMS OF HEAT CONDUCTION

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The article explains the method of solving nonlinear problems of heat conduction with the aid of hybrid computer systems. It examines the possibility of using hybrid systems for realizing the method of optimum dynamic filtration.

For solving nonlinear problems of the theory of fields, approximate analytical and numerical methods are at present widely used; these methods are, as a rule, realized on analog and digital computers. Each of these computer types has its advantages and shortcomings. It is only natural that endeavors are made to combine the rapid action of analog computers (parallel execution of computing operations) with the accuracy, flexibility, and high degree of automation of the computing process on digital computers.

Analog models are divided into passive (analog models) and structural models consisting of active elements based on operational amplifiers. The former realize at once the modeled equation, the latter carry out individual mathematical operations. Structural models are used primarily for solving problems described by ordinary differential equations (problems of control, navigation, some problems of dynamics, etc.). As regards problems of the theory of fields, where, as a rule, equations in partial derivatives are examined (and with finite-difference approximation large bodies of the same type of algebraic equations), these are very efficiently solved with passive models, the processes being described by the same mathematical model as the process in the investigated object. Therefore, in devising hybrid systems for modeling physical fields, it is an advantage to combine a digital computer in particular with that kind of analog computer, when the analog network of resistors may be viewed as an exceedingly suitable and high-speed subprogram for solving systems of algebraic equations (the number of nodes of the network determines the order of the system which can be solved on it). The matter is not fundamentally changed if we view the network as basic processor, and the digital computer (or some other automatic digital device) as a device for specifying the initial data, receiving the results of the solution, or carrying out auxiliary operations in the organization of the iteration process. Experience in the operation of similar hybrid computing systems (HCS) shows that they are 1-2 orders of magnitude faster than the digital computers included in a given hybrid system.

The mathematical model of the nonlinear problem of non-steady-state heat conduction comprises the equation

$$\frac{\partial}{\partial x}\left[\lambda(T)\frac{\partial T}{\partial x}\right]+\frac{\partial}{\partial y}\left[\lambda(T)\frac{\partial T}{\partial y}\right]+\frac{\partial}{\partial z}\left[\lambda(T)\frac{\partial T}{\partial z}\right]=c(T)\rho(T)\frac{\partial T}{\partial t}, \tag{1}$$

the initial $T_{t=0} = f(x, y, z)$, and the boundary conditions:

of the first kind (I)

$$T_s = f(x, y, z, t);$$

of the second kind (II)

$$q_s = f(x, y, z, t);$$

of the third kind (III)

$$\alpha(x, y, z, t)\,[T_s - T_m(t)] = -\lambda(T)\frac{\partial T}{\partial n}; \tag{2}$$

of the fourth kind (IV)

$$T_{1s} = T_{2s};\quad q_{1s} = q_{2s}.$$

When HCS are used for modeling system (1)-(2), the first thing that suggests itself as a matter of course is the automation of Liebmann's method [1] or of the method of successive intervals, which is widely used for solving nonlinear problems of the theory of fields with resistive networks. This is precisely how the analog–digital computer complex (ADCC) "Saturn" [2] is arranged, which is a hybrid computing system (HCS) consisting of the universal computer M-222 and a completely code-controlled network processor "Vega" for 1024 two-coordinate nodes (the "Saturn-2" has 2048 three-coordinate nodes).

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Although the hybrid systems of type ADCC "Saturn" have great possibilities, they are unacceptable to most users because of their high price, which is due to the large number of code-controlled elements of the analog processor.

The price of the system can be substantially reduced by keeping the number of these elements small; this requires that the mathematical model of the phenomenon under examination be correspondingly processed. For instance, Eq. (1) can be transformed into the equation

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{a}\frac{\partial T}{\partial t}-\frac{1}{\lambda}\left(\frac{\partial \lambda}{\partial x}\frac{\partial T}{\partial x}+\frac{\partial \lambda}{\partial y}\frac{\partial T}{\partial y}+\frac{\partial \lambda}{\partial z}\frac{\partial T}{\partial z}\right) \tag{3}$$

or

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{a}\frac{\partial T}{\partial t}-\frac{1}{\lambda}\frac{\partial \lambda}{\partial T}\left[\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2+\left(\frac{\partial T}{\partial z}\right)^2\right], \tag{4}$$

as was done, e.g., by Kozdoba [3].

In Eqs. (3) and (4) there is no nonlinearity in the left-hand sides, and they can be modeled on a code-controlled network with constant structure. The right-hand side of the equation is modeled by additional current which can be introduced into each node of the model, e.g., with the aid of one code-controlled element, a current lead-in. However, it is not easy to calculate the intensity of the led-in current; this is borne out by the form of the right-hand sides of Eqs. (3) and (4).

It is better to change the initial mathematical model with the aid of special transformations, e.g., by using the widely used [4] Kirchhoff substitution:

$$\Theta=\int_0^T \lambda(T)\,dT. \tag{5}$$

This substitution makes it possible to linearize the left-hand side of Eq. (1), and it then assumes the form

$$\frac{\partial^2 \Theta}{\partial x^2}+\frac{\partial^2 \Theta}{\partial y^2}+\frac{\partial^2 \Theta}{\partial z^2}=\frac{1}{a(\Theta)}\frac{\partial \Theta}{\partial t}, \tag{6}$$

and in the steady-state case in general it becomes linear, turning into a Laplace equation. With boundary conditions I and II, which remain linear, the equation can be solved in principle with an analog processor of the HCS without participation of a digital computer, whose function in this case is confined to receiving information from the analog processor and issuing this information in digital form or using it for further calculations. With boundary conditions III, which after transformation (2) with the aid of (5) remain nonlinear:

$$\alpha\,[T_s(\Theta)-T_m]=-\frac{\partial \Theta}{\partial n}, \tag{7}$$

the functions of the digital computer in the HCS are broadened because, in addition to the above operations, it also has to process the signals coming from the boundary nodes of the network according to the left-hand side of Eq. (7), and it has to specify the corresponding currents for these nodes. The solution is effected by iterations, like in Liebmann's method, but when the present method is used, only the parameters of a small part of nodes of the network (nodes at the boundary of the modeled domain) are subject to change, whereas all the other elements of the network maintain their structure unchanged and do not require any corrective action.

The role of the digital computer is even greater in solving non-steady-state problems. In this case it processes signals coming not only from the boundary nodes, but also from internal nodes of the network, and it specifies the current to these nodes in proportion to the right-hand side of Eq. (6). In this case, the code-controlled elements have to be part of the entire structure of the analog processor but their number, in dependence on the procedure used, is only 40 to 20% of the number of elements in the HCS that are similar to the ADCC "Saturn."

Various methods can be used to solve Eq. (6). Depending on the selected procedure, the structures of the analog processor, the share of the digital and analog parts of the HCS in solving the problem, the number of code-controlled elements, and in consequence of all this, the price of the HCS will differ. In [5], various approaches to the realization of the solution of Eq. (6) in a HCS, presented in finite-difference form, were examined; a classification of network processors was presented; the structures of various processors were compared; the possibility was demonstrated of using the principle of superposition for solving nonlinear problems of non-steady-state heat conduction. As a result of the analysis of various versions of analog structures, taking the possibility and price of their realization at the present time into account, it was demonstrated that the most acceptable structure of an analog processor of the HCS is a spatial network of code-uncontrolled resistors with two code-controlled elements in each mode and with twofold use of the principle of superposition. Such an analog processor, which is relatively cheap (half the price indicator of a fully code-controlled structure), is most efficient in regard to informational productivity and the volume of the internal memory of the digital computer, and it takes second place (after the fully code-controlled structure) as regards its speed.

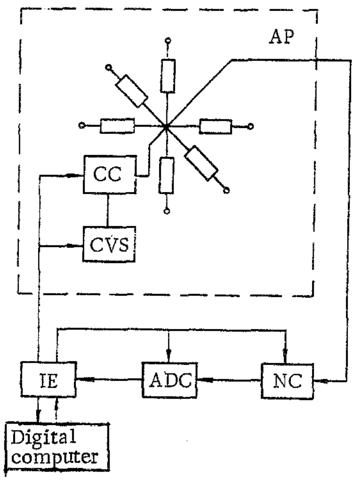

Fig. 1. Block diagram of the analog–digital computer complex.

The above analog structure was made the basis of the analog–digital computer complex "Neptun" [6], designed at the Institute of Problems of Engineering, Academy of Sciences of the Ukr. SSR, which at present is undergoing its debugging tests. The ADCC "Neptun" (Fig. 1) consists of the M-222 digital computer, the network analog processor (AP), the information-exchange unit (IE), the node commutator (NC), and the analog–digital converter (ADC). The network AP contains the network of resistors with 512 nodes and the unit of digital–analog converters, whose part is played by the code-controlled conductances (CC) and voltage sources (CVS). These elements are instrumental in realizing the method of successive intervals [1] in the modification where from iteration to iteration, only the "time-dependent resistance" R_t changes, realized with the aid of the CC, and the voltage supplied by the CVS and corresponding to the value of the function Θ at the preceding step in time. More detailed information on the device and the operating principle of the ADCC "Neptun" can be found in [6-8].

We want to point out that for a number of problems the HCS need not contain a digital computer. A digital computer is necessary in solving a complex of thermophysical problems entailing a large number of algorithmic operations, and where the solutions of individual problems, often following one after the other and influencing each other, as occurred during the design of the ADCC "Neptun," have to be made consistent. When the calculations are simpler, it is advisable to use autonomous systems or HCSs (not connected with a digital computer) which, in addition to AP and a digital computer, contain one or several digital specialized processors (DSP), and also devices for feeding initial data and for the output of the results of the solution. For instance, for solving problems of heat conduction by the above-described method, the most efficient is a three-processor HCS consisting of an AP, a digital computer, and a DSP, where the DSP has the function of organizing an autonomous iteration computing process; this greatly increases the speed of the entire system.

With the ADCC "Neptun," like with a three-processor HCS, inverse problems of heat conduction can be solved. For this, the method [9-11] may be used. However, it is more expedient to connect an additional DSP to the mentioned systems; from the mismatch signal between the specified potential of the node (the analog of the temperature known from the physical experiment) and the potential obtained at this point as a result of modeling, the additional DSP has to calculate the necessary correction of the boundary conditions and issue a command to the AP for their change. Like in the solution of direct problems of heat conduction, the organization of the iteration process at internal nodes remains the function of the digital computer or of the basic DSP.

We will dwell in particular on the use of hybrid systems, especially the ADCC "Neptun," for realizing the solutions of inverse problems of heat conduction by the method of optimum dynamic filtration, which in recent times has been used among other probabilistic methods in solving the examined class of problems [12-14].

The method explained in [15], with all its positive aspects (invariance to different problems of heat conduction, high accuracy of the obtained results, etc.), has nevertheless, like other attempts to use the optimum filter for solving problems of heat conduction, a substantial shortcoming consisting in the fact that the solution of problems with a large number of nodes requires a very large memory capacity and high speed of the computing means, which the present digital computers do not have. On the other hand, the identified parameters (e.g., the heat-transfer coefficients) are, as a rule, affected in the solution of the inverse problem by fully determinate, often bounded zones of the temperature field of the object, i.e., for solving the problem it suffices to refine the temperature only in those zones, whereas the information on the rest of the temperature field may remain more inaccurate. This information (even though inaccurate, nevertheless relatively reliable) can be obtained by modeling on analog devices (HCS based on AP), and the temperatures in zones directly affecting the identification of the boundary conditions will be refined by the digital processor. When the problem is thus stated, the matrices and vectors contained in the filter are of much lower order, and this also obviates the stringent requirements that the memory and the speed of the digital computer have to fulfill; the use of the iteration modification of the filter [16] makes it possible substantially to increase the accuracy of the solution.

NOTATION

T, temperature; λ, thermal conductivity; c, specific heat; ρ, density; t, time; x, y, z, Cartesian coordinates; q, heat flux; α, heat-transfer coefficient; *a*, thermal diffusivity; R, electrical resistance. Subscripts: s, surface; c, medium.

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